Budget Analysis of Drop Size Distribution: A Case Study during SoWMEX/TiMREX

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Introduction

- Former studies
- Research purpose

Data sources

- Instruments
- Case

Methodology

- Interpolate radar data to a Cartesian grid
- Estimate the mean speed of the precipitation system
- Correct observation time difference within a volume scan
- Retrieve the three-dimensional wind
- Retrieve the DSD (drop size distribution)
- Calculate the DSD budget equation

Results

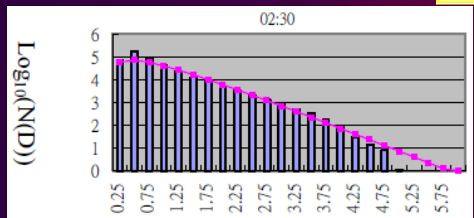
- Budget analysis
- Conclusions

Wind retrieval:

Armijo (1969)	Multiple Doppler radar synthesis
Gal-Chen (1982)	Single radar - 3DVAR in a moving frame of reference
Sun et al. (1991)	Single radar - 4DVAR using a dynamic model
Shapiro et al. (1995)	Single radar - Real case studies based on GC82
Zhang & Gal-Chen (1996)	
Liou (1999, 2002, 2007)	Single radar – works on evaluating and improving ZG96

DSD retrieval:

Marshall & Palmer (1948)	Exponential DSD $N(D_e) = N_0 e^{-\Lambda D_e}$
Ulbrich (1983)	Gamma DSD $N(D_e) = N_0 D_e^{\mu} e^{-\Lambda D_e}$
Zhang (2001)	



Gorgucci et al. (2000, 2002)	β method
Zhang et al. (2001)	μ - Λ relation
Brandes et al. (2002, 2003)	Constrained gamma method
Ji (2005)	Substitute K_{dp} for Z_{hh} in the constrained gamma method

Introduction

Research Purpose

$$\frac{dN_i}{dt} = \frac{\partial N_i}{\partial t} + u\frac{\partial N_i}{\partial x} + v\frac{\partial N_i}{\partial y} + (w - v_t)\frac{\partial N_i}{\partial z} = C_i^n + C_i^d + C_i^c + C_i^b$$

N_i Drop number concentration of the i-th size bin

$\frac{dN_i}{dt}$ Total derivative of N_i, standing for the rate of change of N_i following the drop motion

$\frac{\partial N_i}{\partial t}$ Local derivative of N_i, standing for the rate of change of N_i at a fixed point

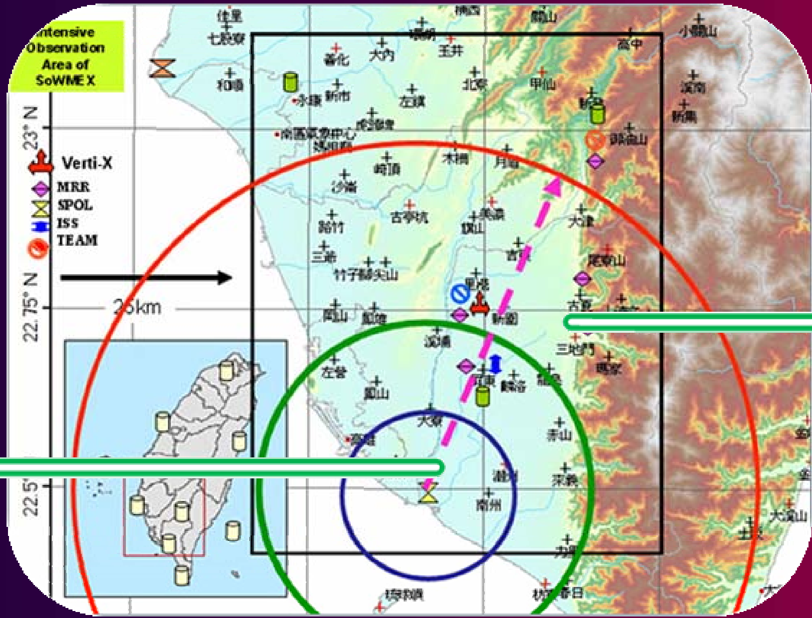
$u\frac{\partial N_i}{\partial x} + v\frac{\partial N_i}{\partial y} + w\frac{\partial N_i}{\partial z}$ Advection, standing for the rate of change of N_i due to air velocity

$-v_t\frac{\partial N_i}{\partial z}$ Sedimentation, standing for the rate of change of N_i due to terminal velocity

$C_i^n + C_i^d + C_i^c + C_i^b$ Microphysics, standing for the rate of change of N_i due to nucleation, vapor diffusion, collision coalescence and breakup

Data sources *Instruments*

NCAR SPOL



SoWMEX/TiMREX
(2008.5.15~6.30)

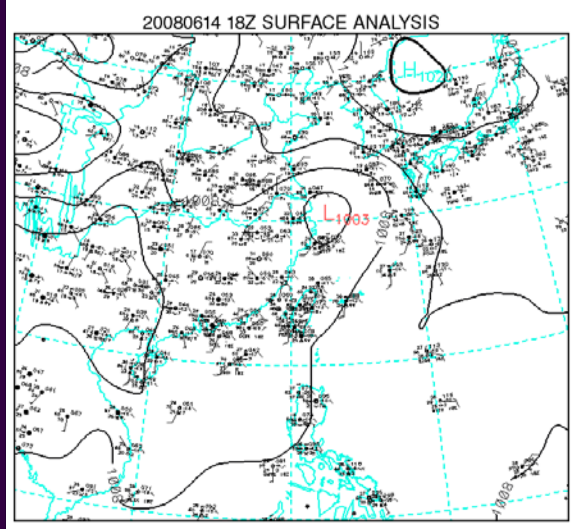
NCU 2DVD

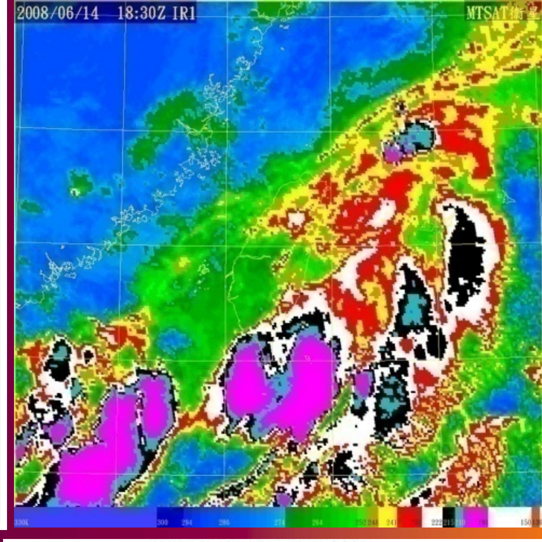
NCAR SPOL		NCU 2DVD
120.43E 22.53N	location	120.62E 22.74N
6.14 180924~184654 UTC	analysis period	6.14 000000~240000 UTC
Z_{hh}, v_r, Z_{dr}, t_g	wanted parameters	μ , Λ
11 volume scans 10 sweeps per volume azimuth 105~230 elevation 1.5~11.8	strategy	accumulate every 3 minutes 214 rainy accumulations

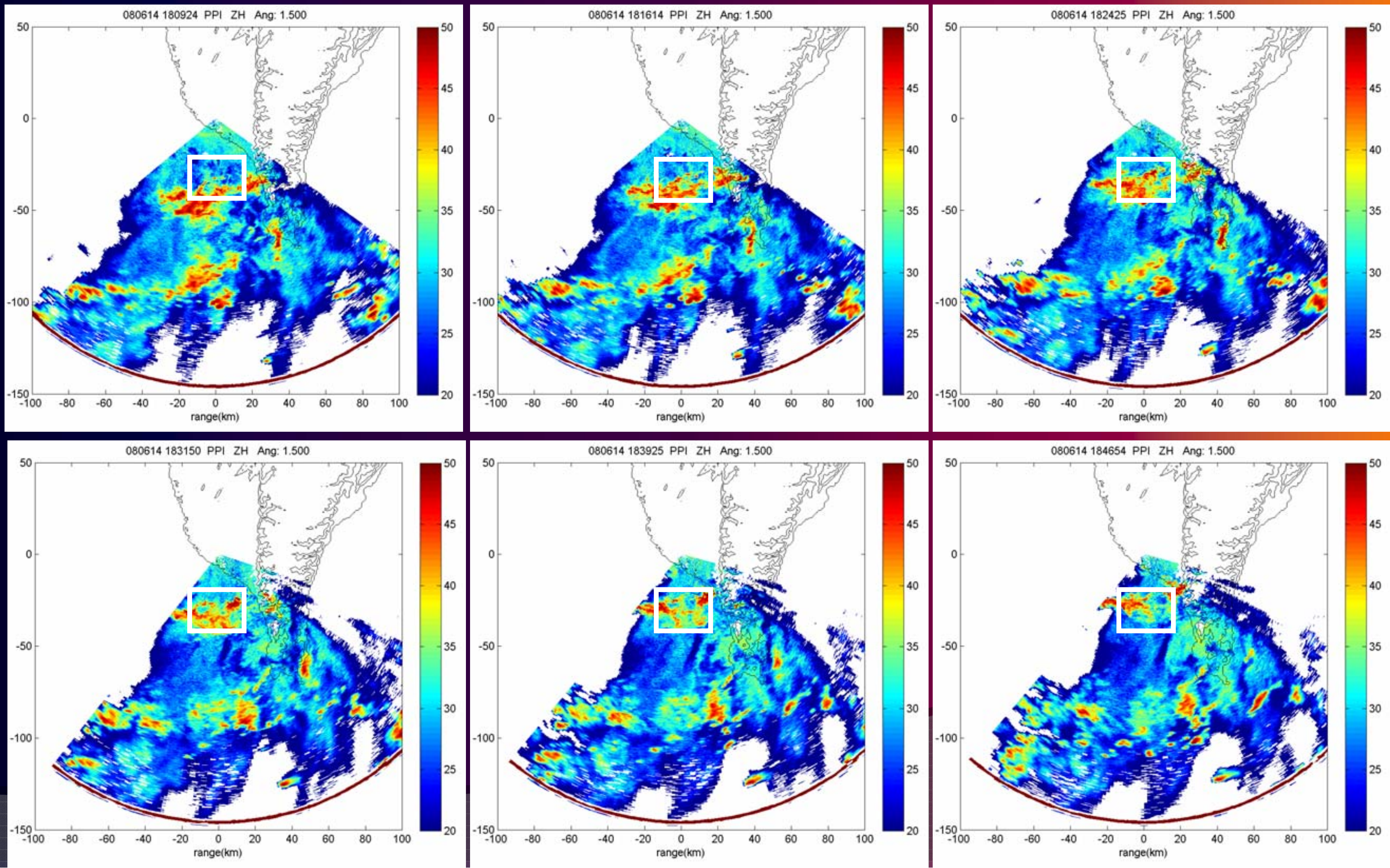
Data sources

Case

	surface map 1800	MTSAT IR Color 1830
Z*hh* PPI 180924	Z*hh* PPI 181614	Z*hh* PPI 182425
Z*hh* PPI 183150	Z*hh* PPI 183925	Z*hh* PPI 184654







Data sources *Case*

Z_{hh} CAPPI (1.75 km)

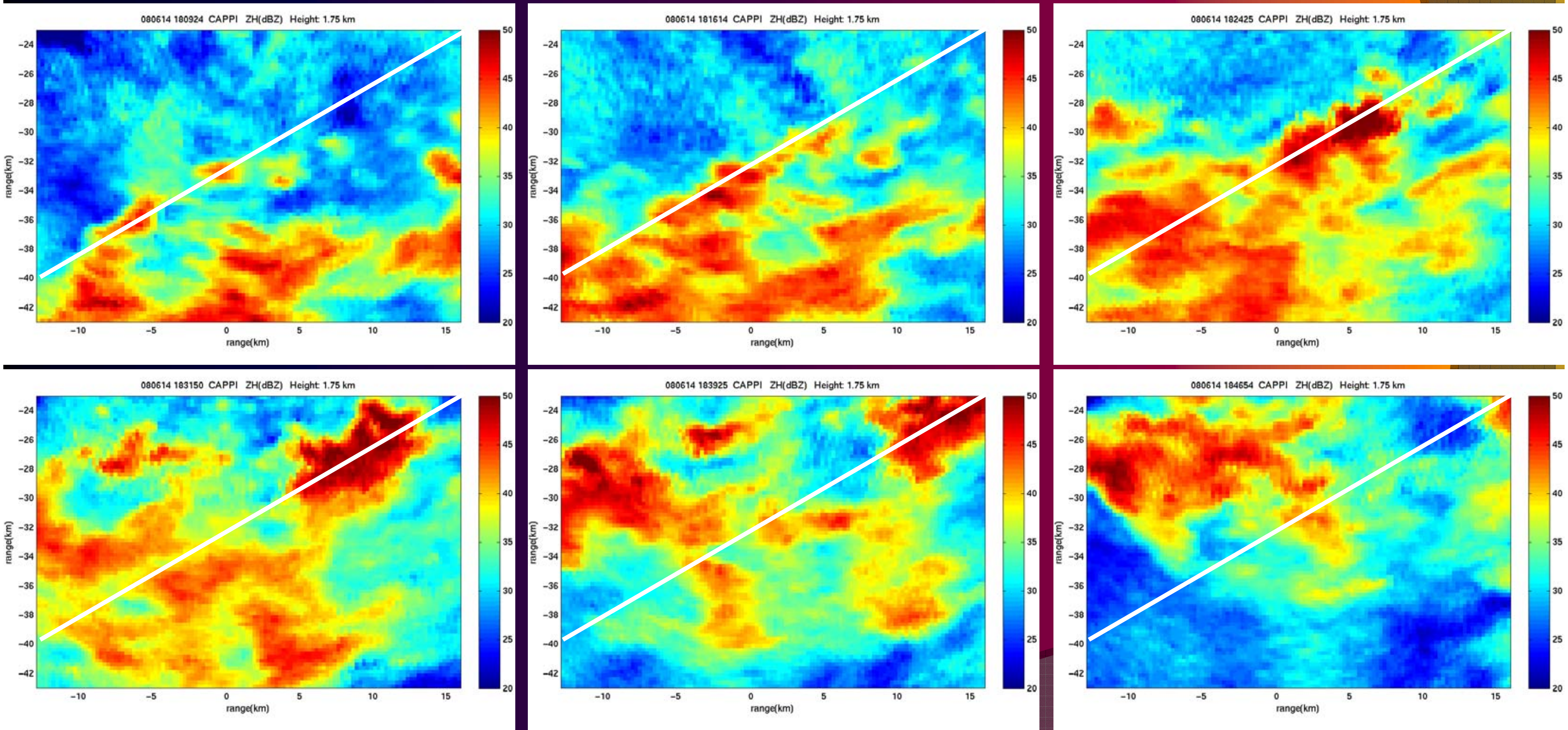
180924	181614	182425
183150	183925	184654

Domain of analysis:

x = -13 ~ 16 km (res. = 0.25 km)

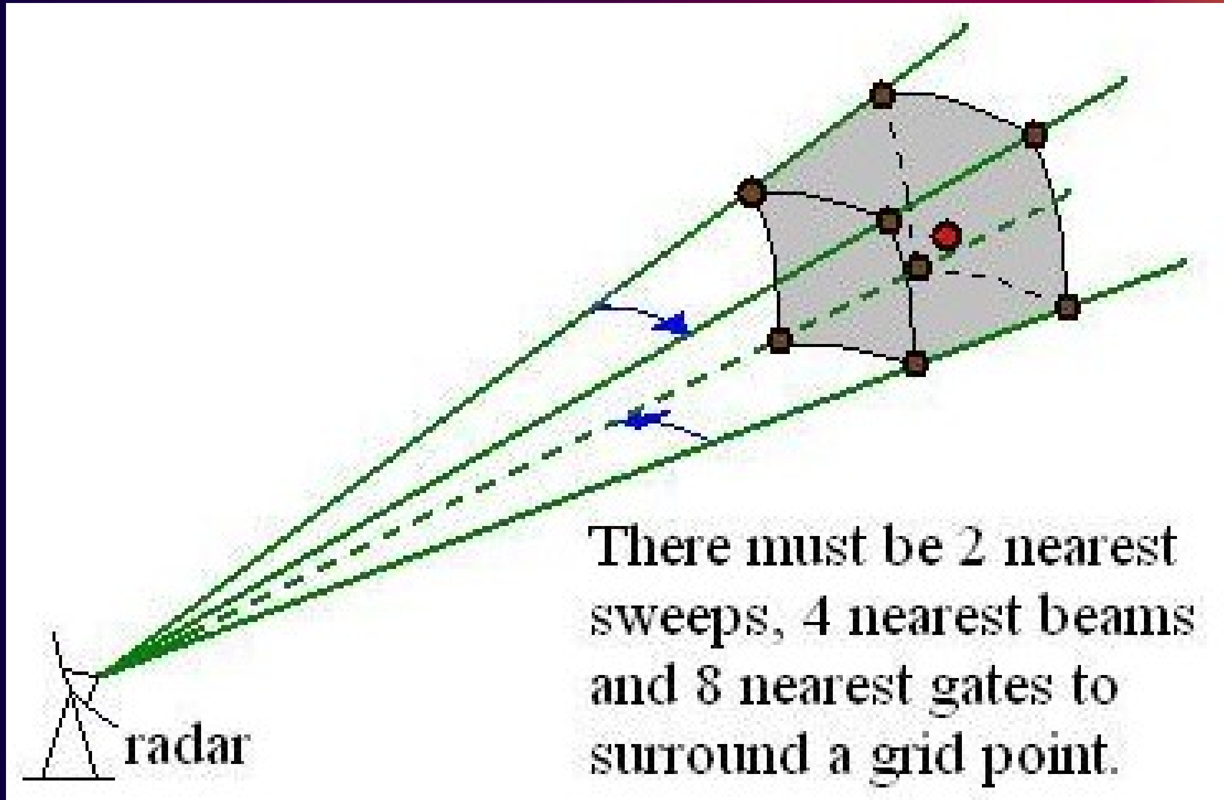
y = -43 ~ -23 km (res. = 0.25 km)

z = 1.75 ~ 3.75 km (res. = 0.25 km)



Methodology

Interpolate radar data to a Cartesian grid



Z_{hh} v_r Z_{dr} t_g

K_{dp} ρ_{hv}

$$A_{int} = \frac{\sum \frac{1}{r_k} A_k}{\sum \frac{1}{r_k}}$$

Methodology *Estimate the mean speed of the precipitation system*

Using the method of Liou (2007) based on an optimal moving frame of reference:

$$J_M = \frac{1}{2}\sum_i\sum_j\sum_k\sum_t \alpha_1 (T_1)^2 + \frac{1}{2}\sum_k \alpha_2 \left[(T_{2\_1})^2 + (T_{2\_2})^2\right]$$

$$T_1 = \frac{\partial\eta}{\partial t} + U(z)\frac{\partial\eta}{\partial x} + V(z)\frac{\partial\eta}{\partial y} + (W + V_T)\frac{\partial\eta}{\partial z}$$

$$T_{2\_1} = \frac{\partial^2 U}{\partial z^2} \qquad T_{2\_2} = \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial J_M}{\partial U} = \sum_i\sum_j\sum_t \left(\alpha_1 T_1 \frac{\partial\eta}{\partial x}\right) + \alpha_2 \frac{\partial^2 T_{2\_1}}{\partial z^2}$$

$$\frac{\partial J_M}{\partial V} = \sum_i\sum_j\sum_t \left(\alpha_1 T_1 \frac{\partial\eta}{\partial y}\right) + \alpha_2 \frac{\partial^2 T_{2\_2}}{\partial z^2}$$

$$\frac{\partial J_M}{\partial W} = \sum_i\sum_j\sum_k\sum_t \left(\alpha_1 T_1 \frac{\partial\eta}{\partial z}\right)$$

Optimal U(z), V(z) and W are found when J_M is minimized after iterations.

Methodology

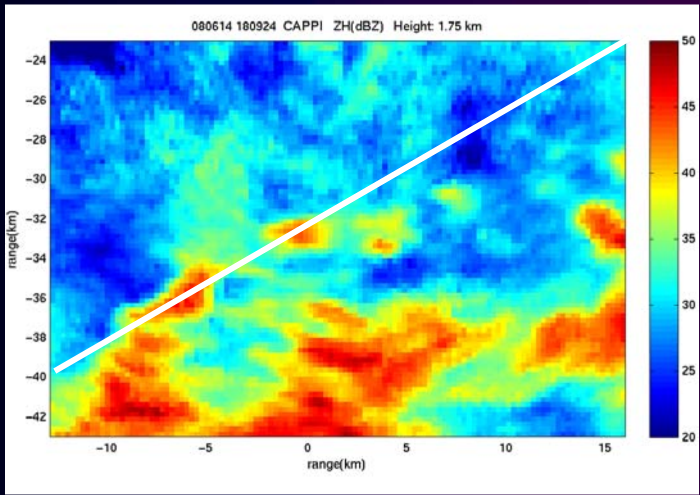
Estimate the mean speed of the precipitation system

Start time of volume 1 (UTC)	Start time of volume 2 (UTC)	Time interval (sec)	U (m/sec)	V (m/sec)	Total speed (m/sec)
180924	181156	152	11.09249	6.140811	12.67883
181156	181614	258	11.86296	6.612912	13.58162
181614	181846	152	11.35029	6.530509	13.09491
181846	182425	339	11.53859	7.149321	13.57394
182425	182703	158	11.48129	7.299694	13.60535
182703	183150	287	11.01013	7.195235	13.15273
183150	183428	158	11.43562	6.648122	13.22766
183428	183925	297	10.19898	6.403155	12.04241
183925	184212	167	10.28821	6.252707	12.03926
184212	184654	282	10.76621	6.379809	12.51452

Objectives: 1. To correct observation time difference within a volume scan.
2. To retrieve the three-dimensional wind.

Methodology

Correct observation time difference within a volume scan

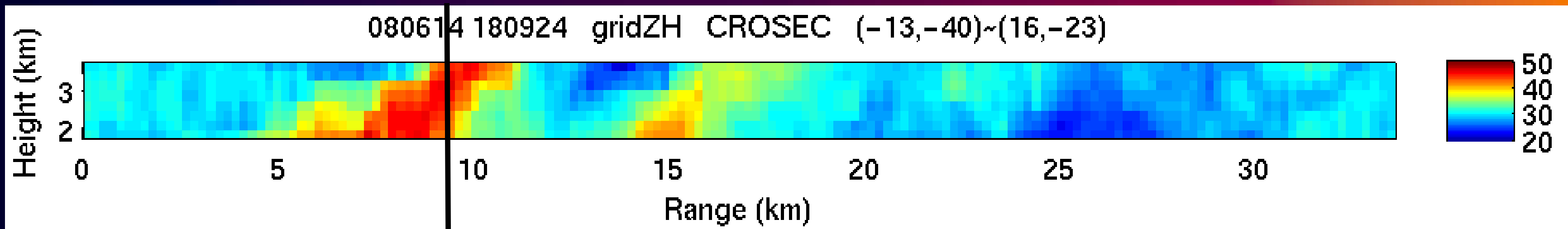


1. Shift parameters on the grid back to where they should have been at the same time (the start time of the volume scan).

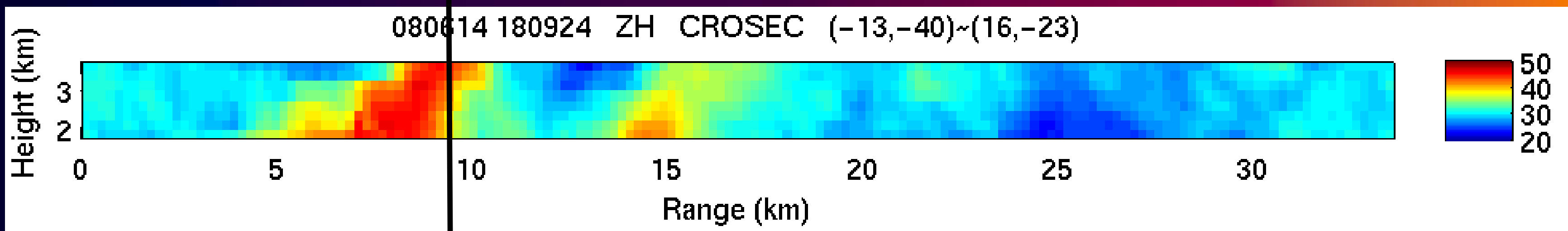
$$x_c = x - U \cdot t_g$$
$$y_c = y - V \cdot t_g$$

2. Re-interpolate data to the Cartesian grid.

Before correction:



After correction:



Methodology

Retrieve the three-dimensional wind

Using the method of Liou (2007) based on an optimal moving frame of reference:

$$x' = x - U(t - t_0)$$

$$y' = y - V(t - t_0)$$

$$z' = z - W(t - t_0)$$

$$t' = t$$

$$u' = u - U$$

$$v' = v - V$$

$$w' = w - W$$

$$J = \frac{1}{2}\sum_i \sum_j \sum_k \sum_t \left[\beta_1 J_1^2 + \beta_2 J_2^2 + \beta_3 J_3^2 + \beta_4 \left(J_{4\_1}^2 + J_{4\_2}^2 + J_{4\_3}^2\right)^2 + \beta_5 J_5^2 + \beta_6 \left(J_{6\_1}^2 + J_{6\_2}^2 + J_{6\_3}^2\right)^2\right]$$

$$J_1 = \frac{\partial \eta'}{\partial t'} + u'\frac{\partial \eta'}{\partial x'} + v'\frac{\partial \eta'}{\partial y'} + (w' + V_T)\frac{\partial \eta'}{\partial z'}$$

$$J_2 = v_r' - u'\frac{x'}{r'} - v'\frac{y'}{r'} - w'\frac{z'}{r'}$$

$$J_3 = \frac{\partial \rho u'}{\partial x'} + \frac{\partial \rho v'}{\partial y'} + \frac{\partial \rho w'}{\partial z'}$$

$$J_{4\_1} = \frac{\partial w'}{\partial y'} - \frac{\partial v'}{\partial z'} \qquad J_{4\_2} = \frac{\partial u'}{\partial z'} - \frac{\partial w'}{\partial x'} \qquad J_{4\_3} = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}$$

$$J_5 = (\bar{u}' - u'_{VAD})^2 + (\bar{v}' - v'_{VAD})^2$$

$$J_{6\_1} = \nabla_H^2 u' \qquad J_{6\_2} = \nabla_H^2 v' \qquad J_{6\_3} = \nabla_H^2 w'$$

$$\frac{\partial J}{\partial u'} = \sum_t \left[\beta_1 J_1 \frac{\partial J_1}{\partial x'} - \beta_2 J_2 \frac{x'}{r'} - \rho\beta_3 \frac{\partial J_3}{\partial x'} + \beta_4\left(\frac{\partial J_{4\_3}}{\partial y'} - \frac{\partial J_{4\_2}}{\partial z'}\right) + \beta_6 \nabla_H^2(\nabla_H^2 u')\right]$$

$$\frac{\partial J}{\partial v'} = \sum_t \left[\beta_1 J_1 \frac{\partial J_1}{\partial y'} - \beta_2 J_2 \frac{y'}{r'} - \rho\beta_3 \frac{\partial J_3}{\partial y'} + \beta_4\left(\frac{\partial J_{4\_1}}{\partial z'} - \frac{\partial J_{4\_3}}{\partial x'}\right) + \beta_6 \nabla_H^2(\nabla_H^2 v')\right]$$

$$\frac{\partial J}{\partial w'} = \sum_t \left[\beta_1 J_1 \frac{\partial J_1}{\partial z'} - \beta_2 J_2 \frac{z'}{r'} - \rho\beta_3 \frac{\partial J_3}{\partial z'} + \beta_4\left(\frac{\partial J_{4\_2}}{\partial x'} - \frac{\partial J_{4\_1}}{\partial y'}\right) + \beta_6 \nabla_H^2(\nabla_H^2 w')\right]$$

Optimal u’, v’, w’ and hence u, v, w are found when J is minimized after iterations.

Methodology *Retrieve the DSD*

Seliga et al. (1976) and Ulbrich (1983):

$$Z_{hh} \equiv 10\log\left[\frac{\lambda^4}{\pi^5|K|^2}\int \sigma_{hh} N(D_e)dD_e\right] = 10\log\left[\frac{16\pi^2}{9|K|^2}N_0\int D_e^{\,6}\left|\frac{m^2-1}{4\pi+(m^2-1)P'}\right|^2 D_e^{\,\mu}e^{-\Lambda D_e}dD_e\right]$$

$$Z_{vv} \equiv 10\log\left[\frac{\lambda^4}{\pi^5|K|^2}\int \sigma_{vv} N(D_e)dD_e\right] = 10\log\left[\frac{16\pi^2}{9|K|^2}N_0\int D_e^{\,6}\left|\frac{m^2-1}{4\pi+(m^2-1)P}\right|^2 D_e^{\,\mu}e^{-\Lambda D_e}dD_e\right]$$

function of r, N_0, μ and Λ

$$Z_{dr} = 10\log\frac{\int D_e^{\,6}\left|\frac{m^2-1}{4\pi+(m^2-1)P'}\right|^2 D_e^{\,\mu}e^{-\Lambda D_e}dD_e}{\int D_e^{\,6}\left|\frac{m^2-1}{4\pi+(m^2-1)P}\right|^2 D_e^{\,\mu}e^{-\Lambda D_e}dD_e}$$

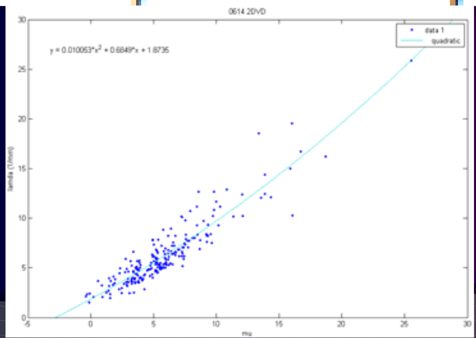
function of r, μ and Λ

Brandes et al. (2002):

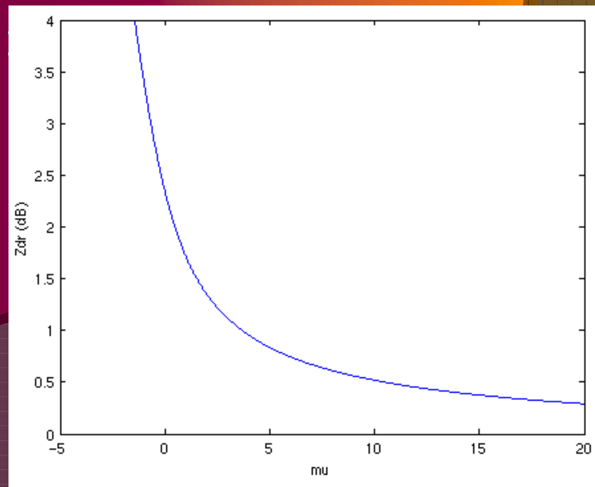
$$r = 0.9951 + 0.02510D_e - 0.03644D_e^{\,2} + 0.005030D_e^{\,3} - 0.0002492D_e^{\,4}$$

NCU 2DVD (2008.6.14):

$$\Lambda = 0.010053\mu^2 + 0.6849\mu + 1.8735$$



Z_{dr} vs. μ :



Methodology

Calculate the DSD budget equation

$$\frac{dN_i}{dt} = \frac{\partial N_i}{\partial t} + u\frac{\partial N_i}{\partial x} + v\frac{\partial N_i}{\partial y} + (w - v_t)\frac{\partial N_i}{\partial z} = C_i^n + C_i^d + C_i^c + C_i^b$$

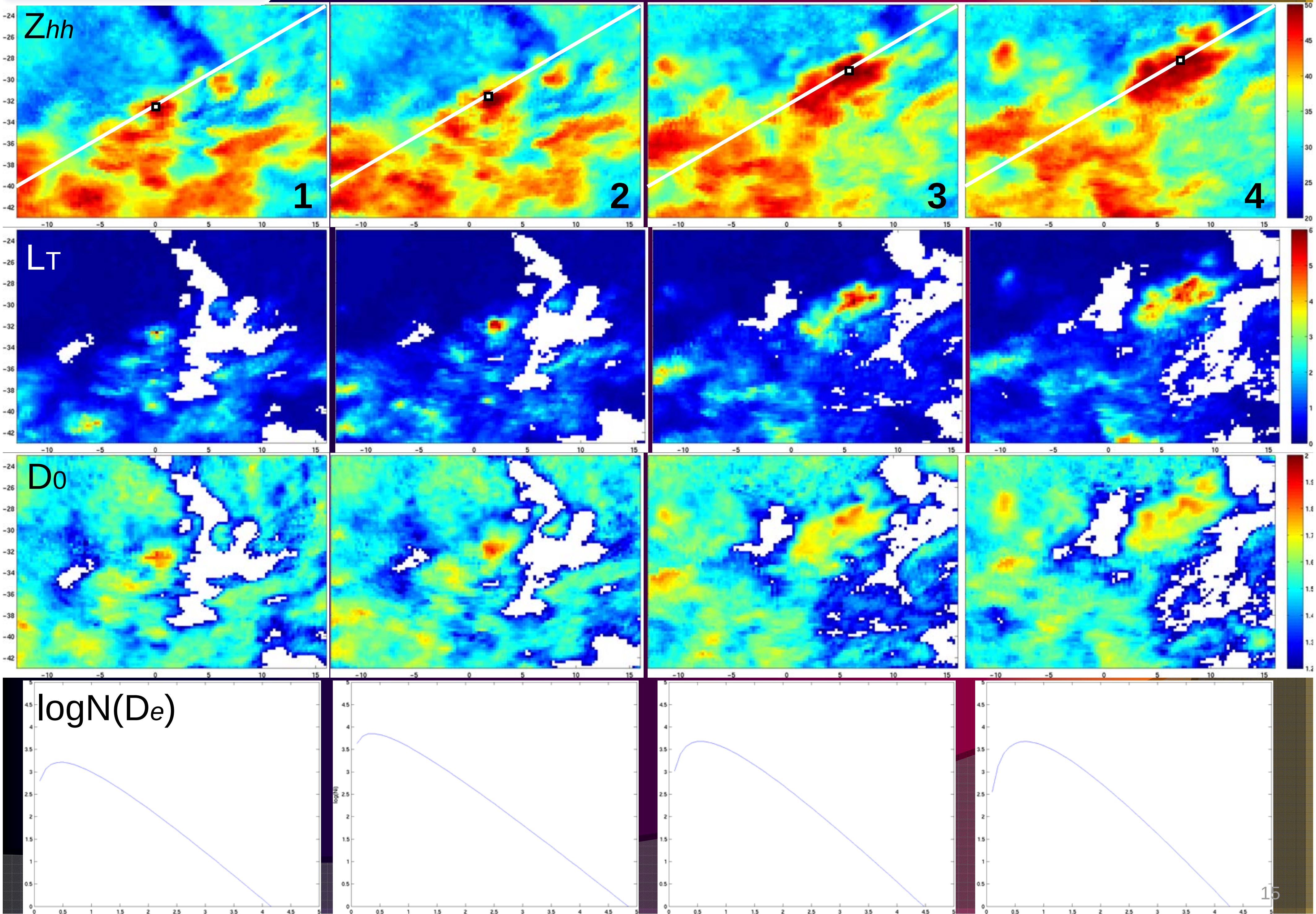
Atlas et al. (1977): $v_t(D_e) = 3.78 D_e^{0.67}$

Beard (1985): $v_t(D_e) = \left(\frac{\rho_0}{\rho}\right)^{0.375+0.025D_e} 3.78 D_e^{0.67}$

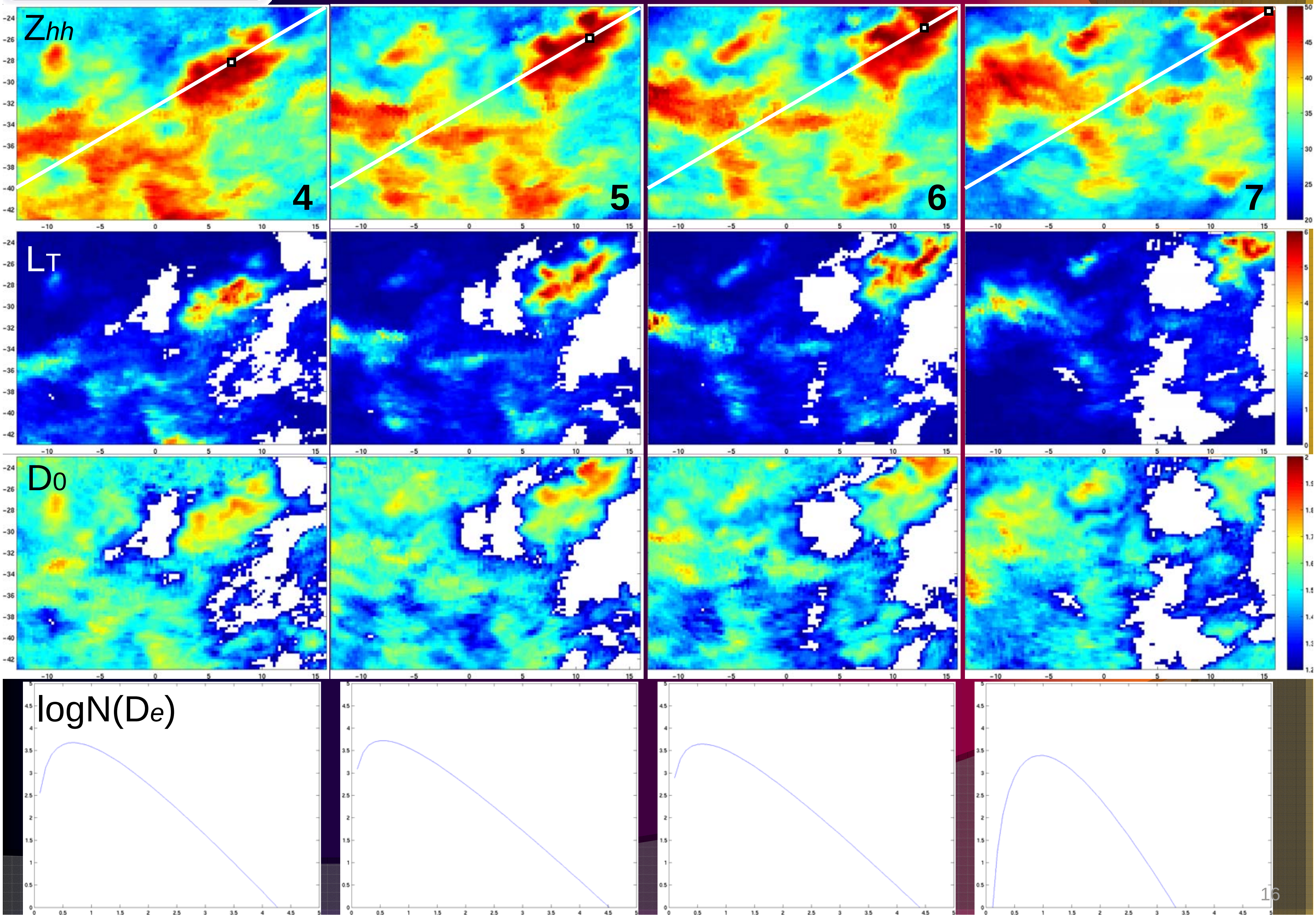
$\frac{dN_i}{dt}$, $\frac{\partial N_i}{\partial t}$, $u\frac{\partial N_i}{\partial x}$, $v\frac{\partial N_i}{\partial y}$, $w\frac{\partial N_i}{\partial z}$, $-v_t\frac{\partial N_i}{\partial z}$ × $10^{-3}\left(\frac{\pi}{6}\rho_w D_e^{3}\right)$ = L_i^{tot}, L_i^{loc}, L_i^{u}, L_i^{v}, L_i^{w}, L_i^{sed}

Results

Budget analysis 1614 1846 2425 2703 2.75 km CAPPI



Results *Budget analysis* 2703 3150 3428 3925 2.75 km CAPPI



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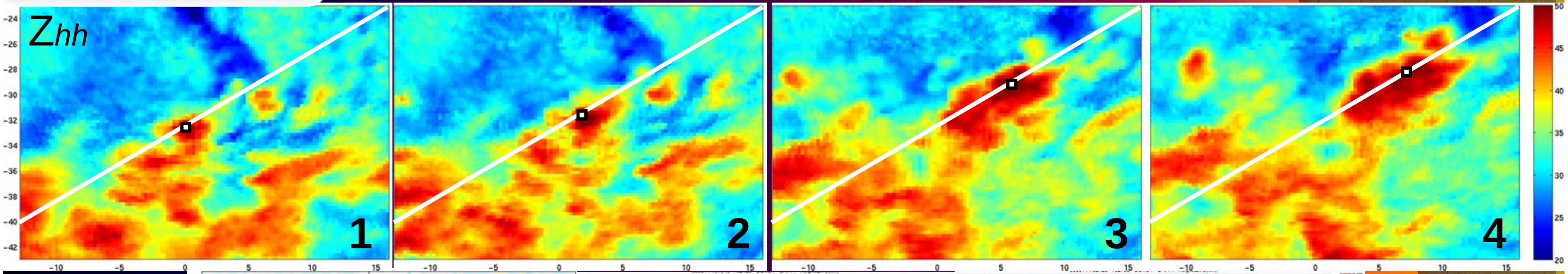

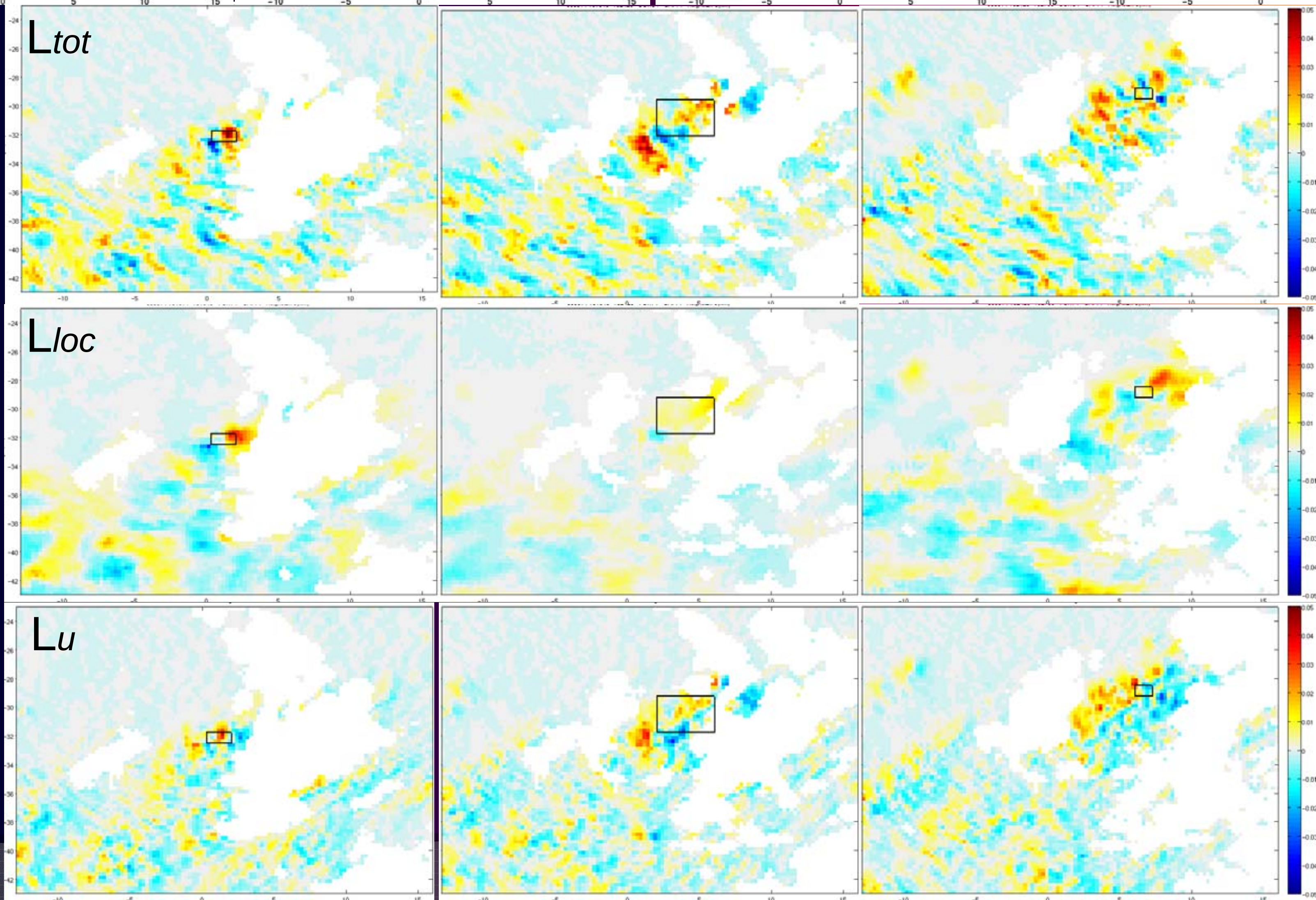

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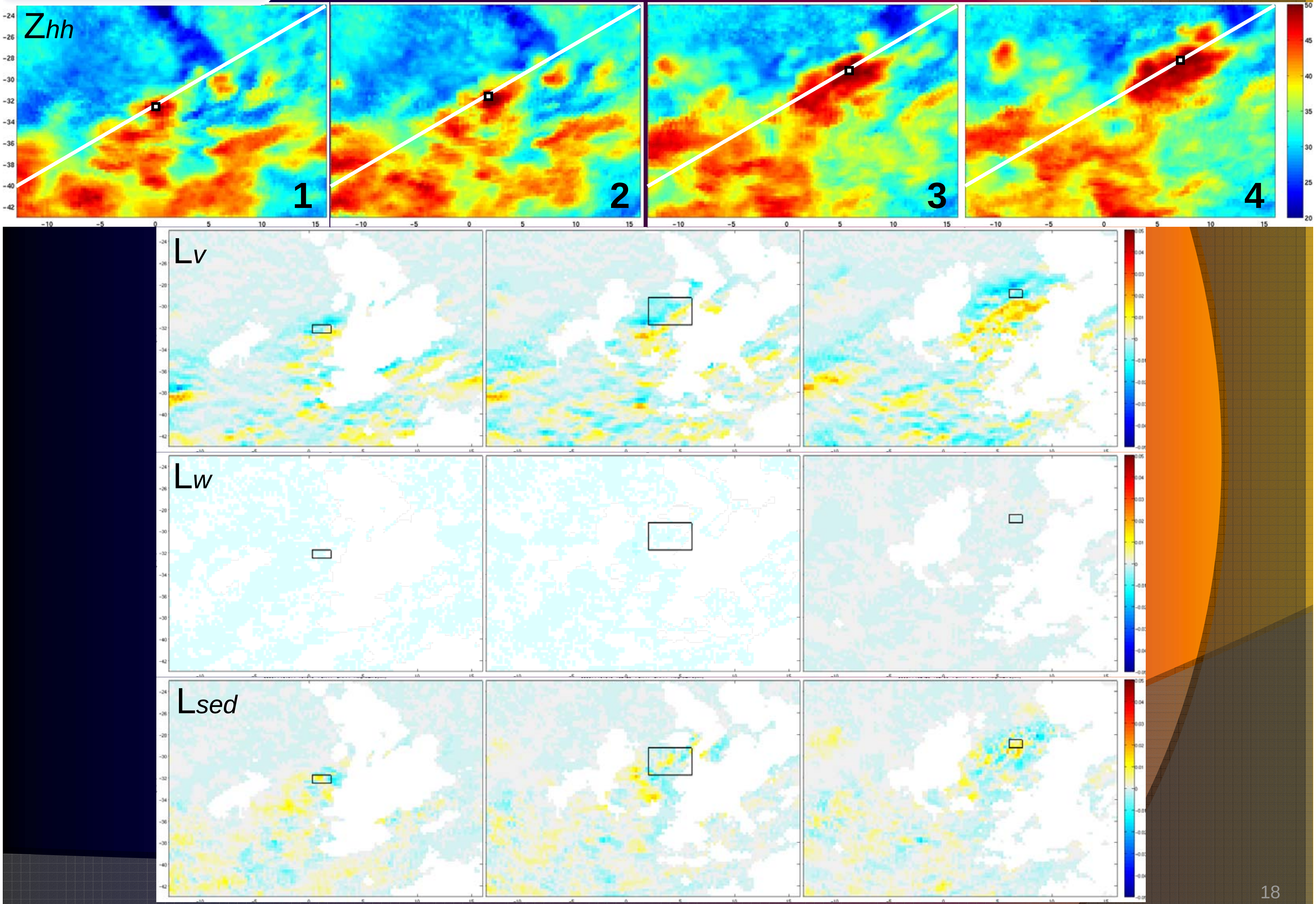
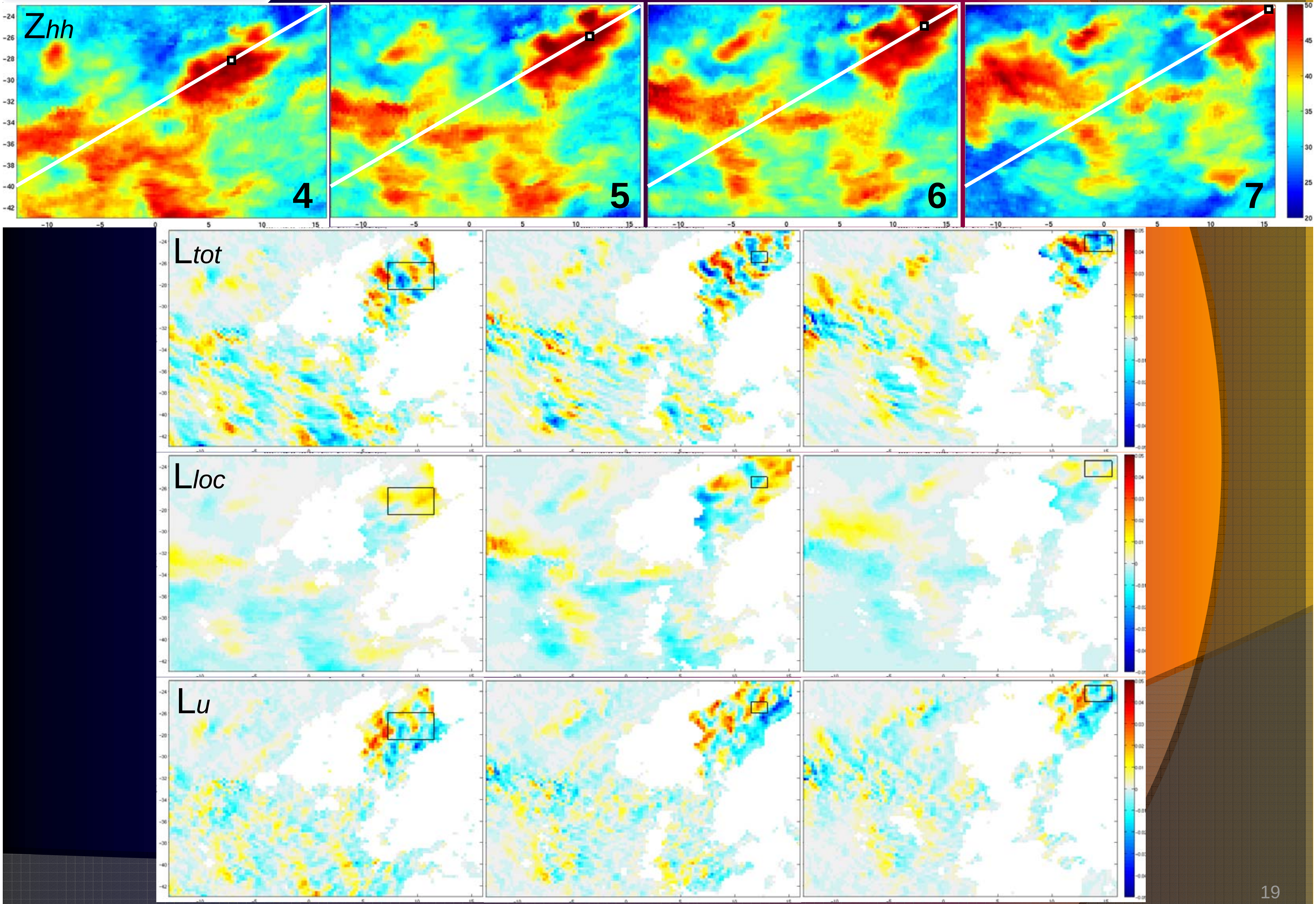
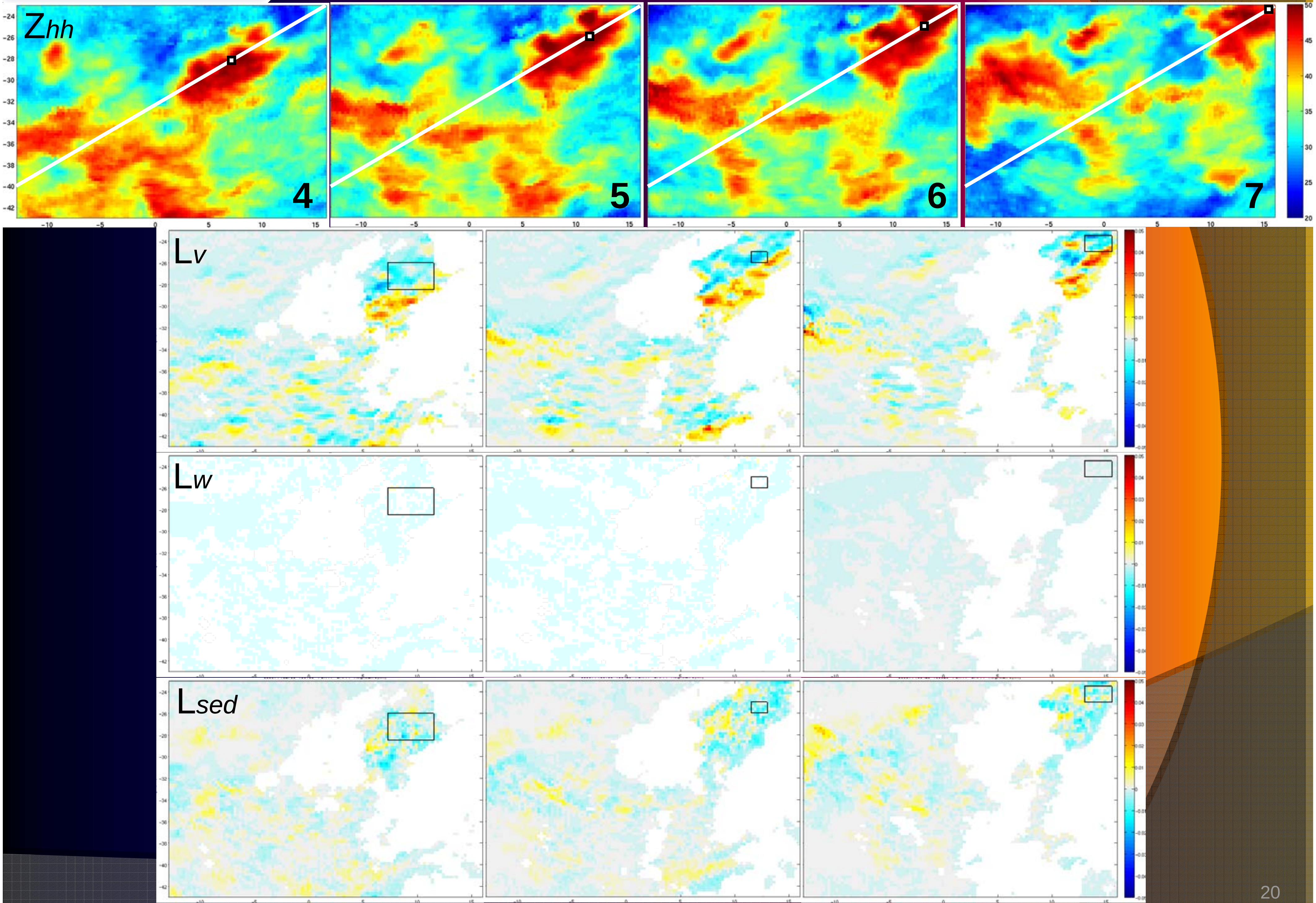


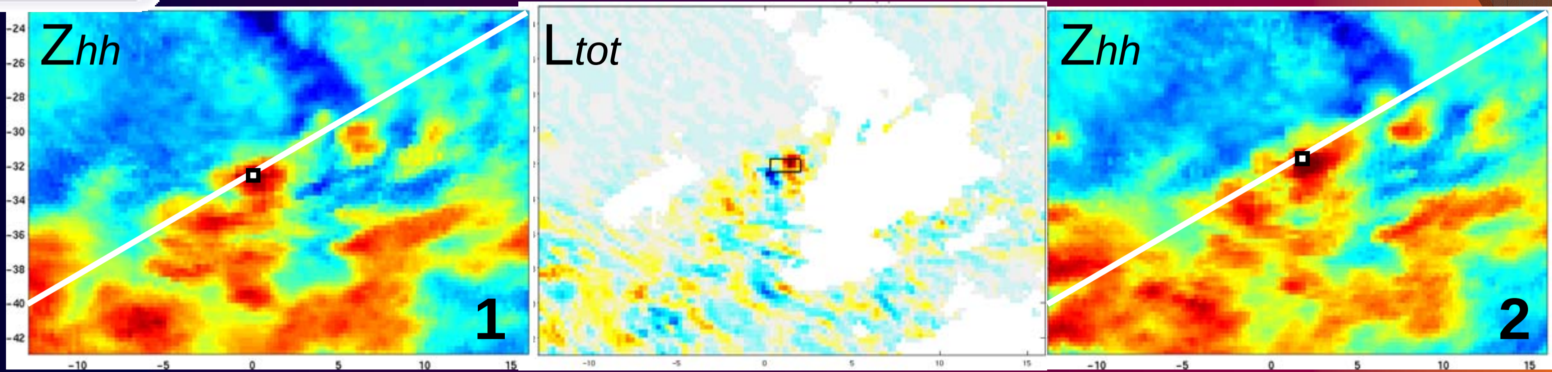
Budget analysis 2703 3150 3428 3925 2.75 km CAPPI

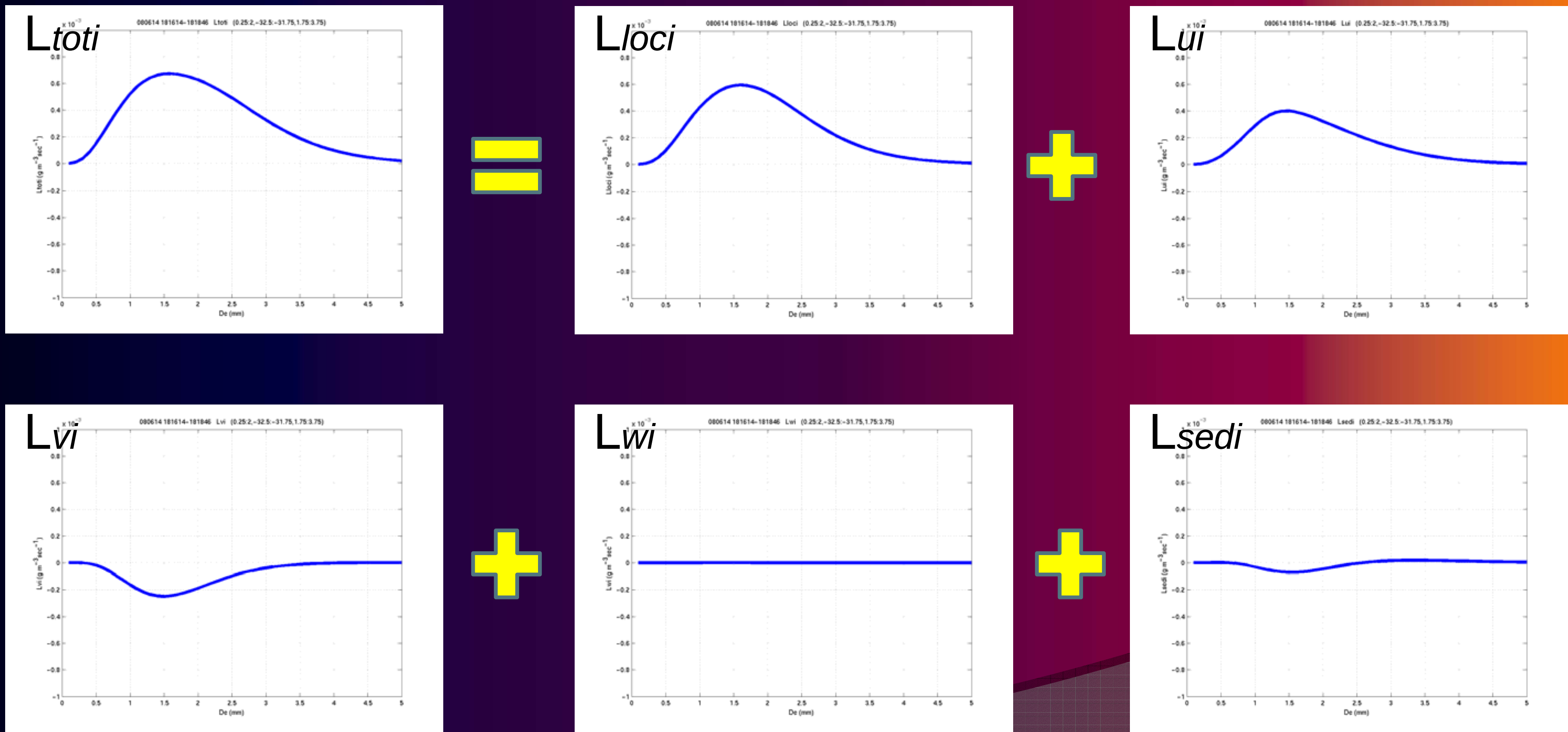


Results *Budget analysis* 2703 3150 3428 3925 2.75 km CAPPI

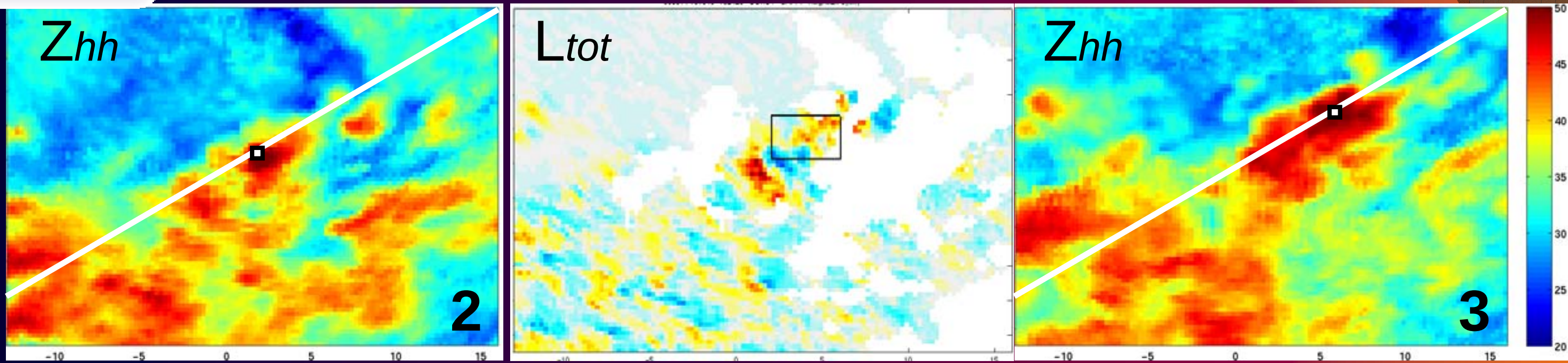


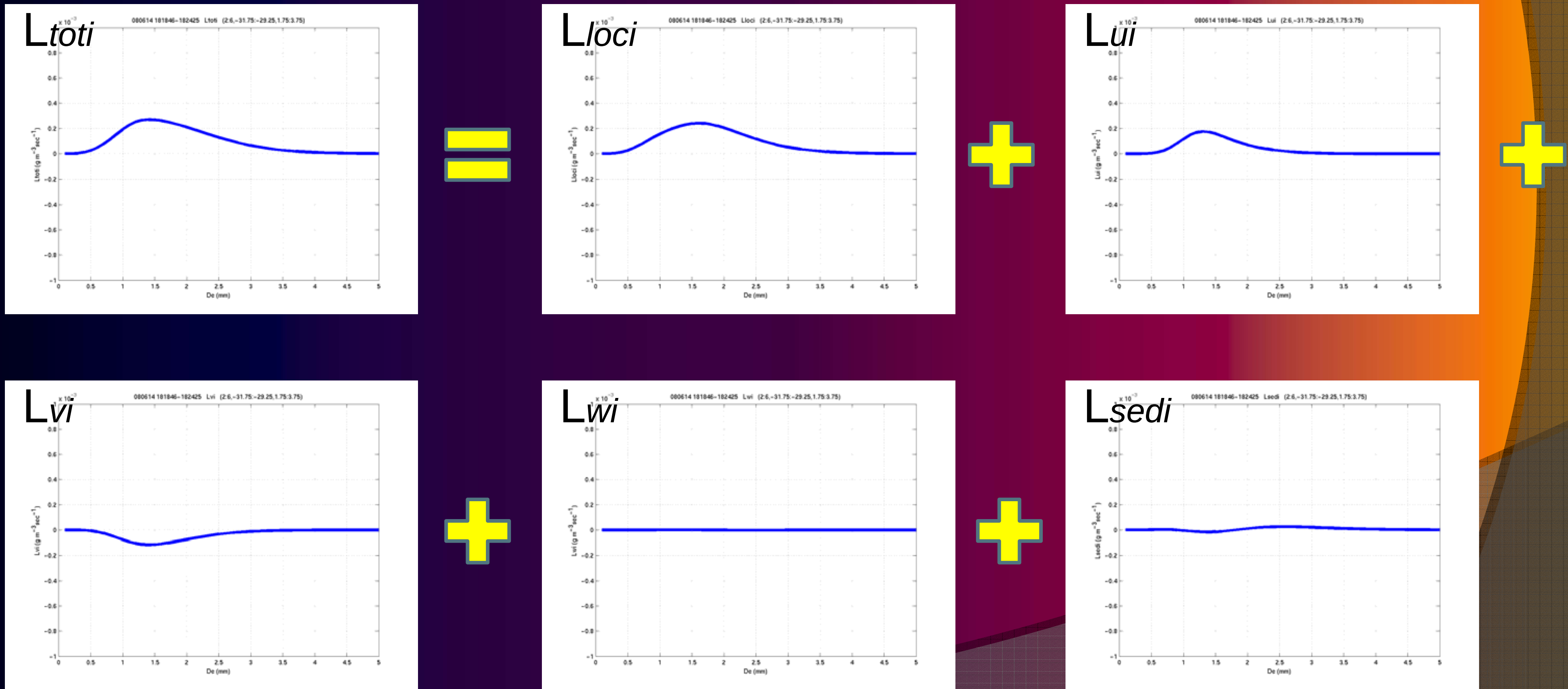
Results *Budget analysis* **1614 1846** 2425 2703 Intensifying stage



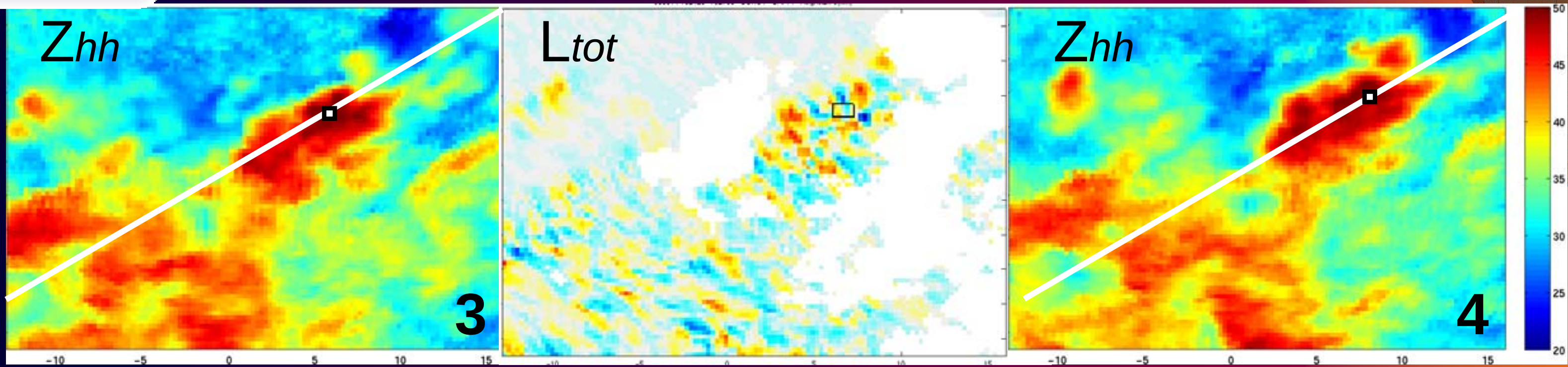


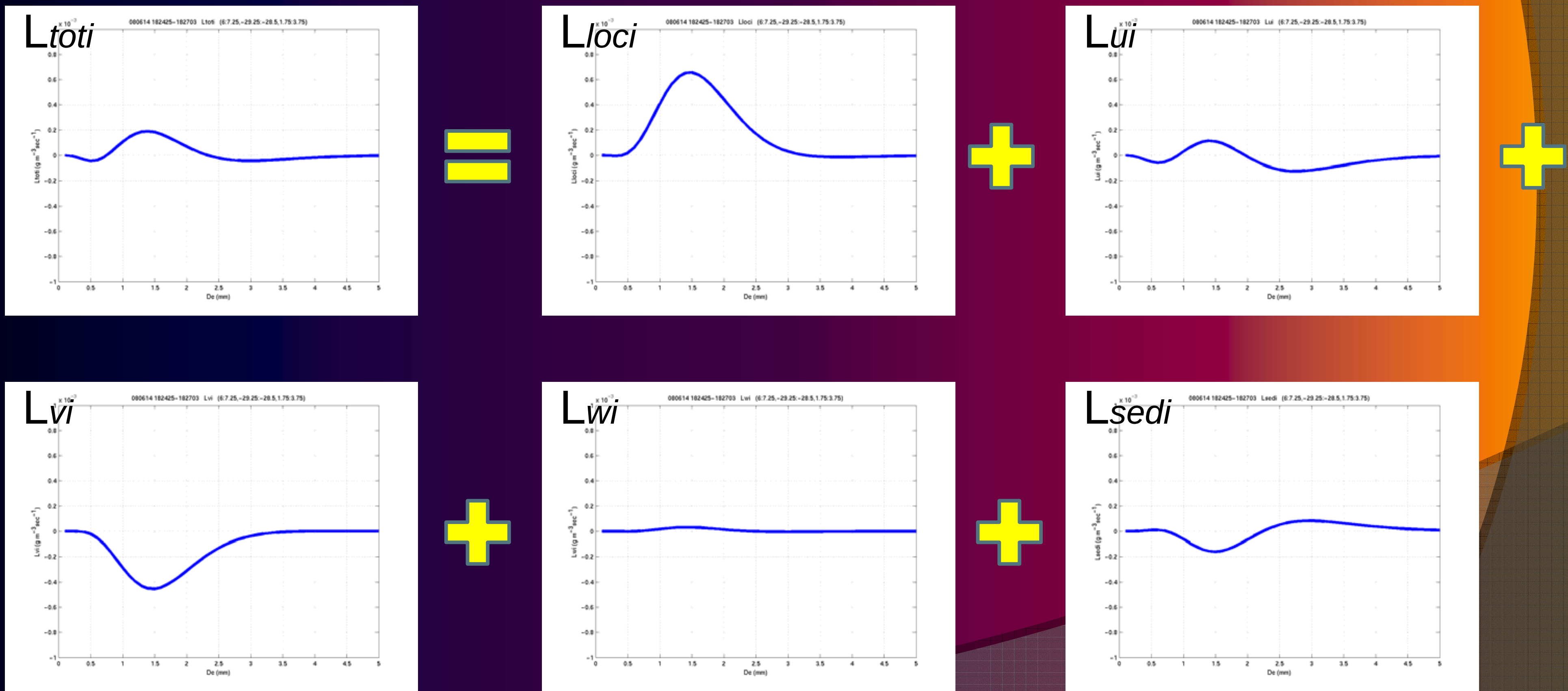
Results *Budget analysis* 1614 **1846 2425** 2703 Intensifying stage



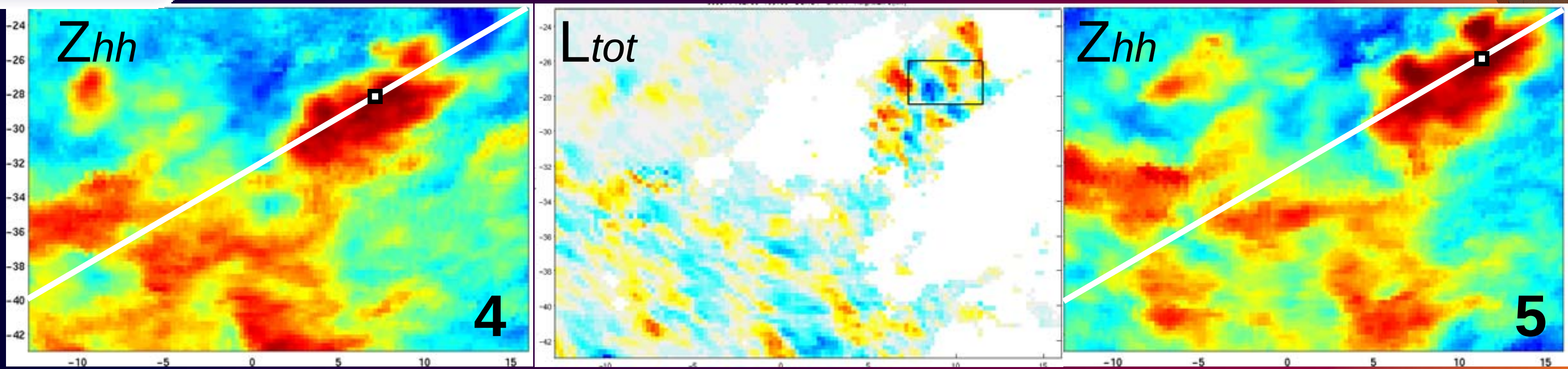


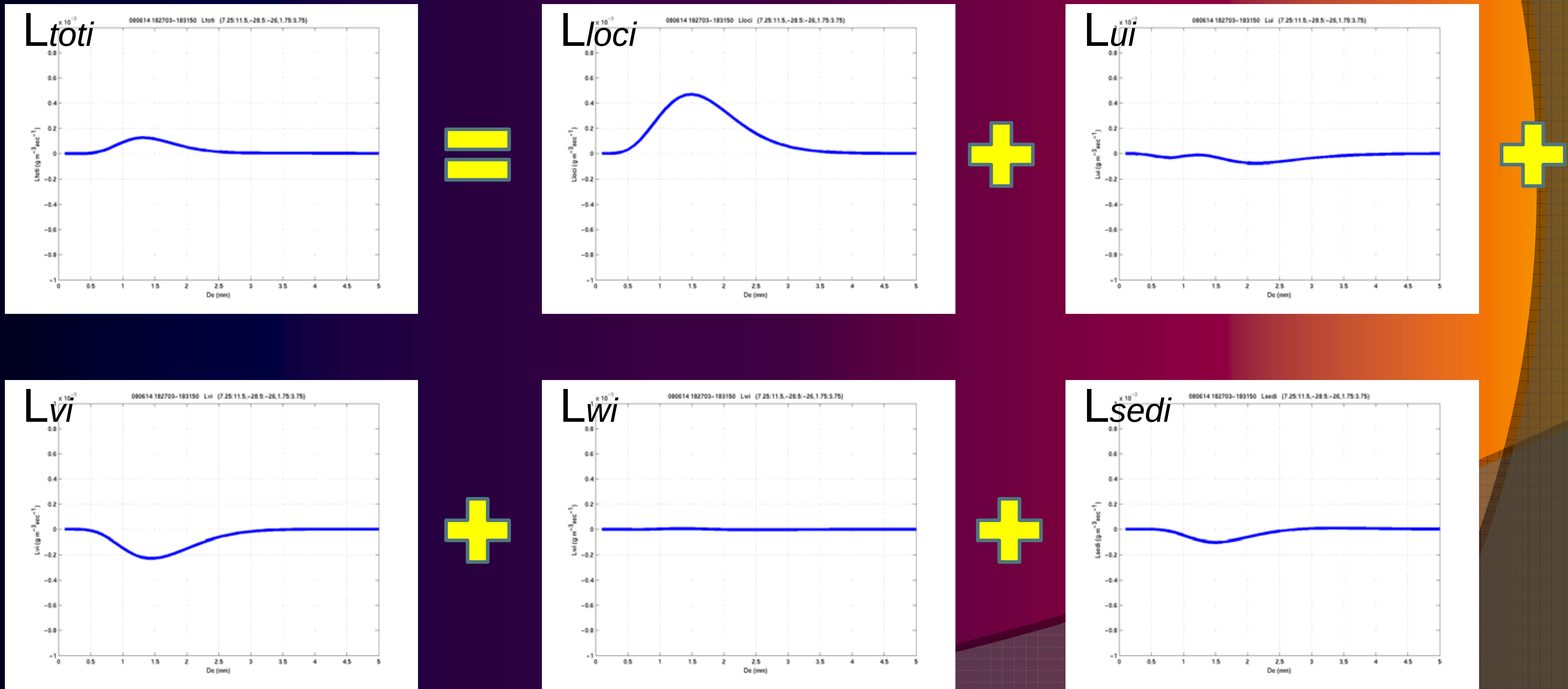
Results *Budget analysis* 1614 1846 **2425 2703** Mature stage I





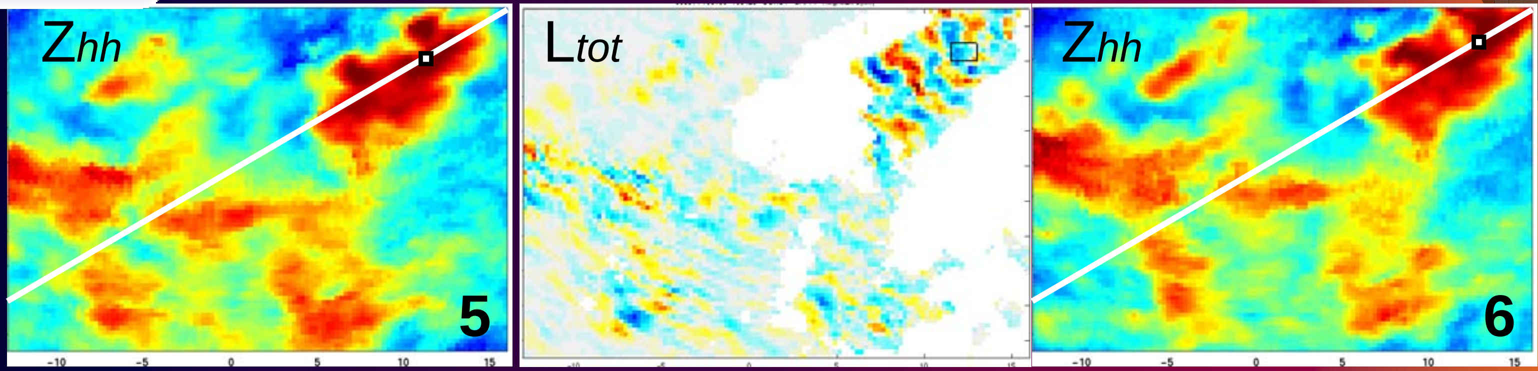
Results *Budget analysis* **2703 3150** 3428 3925 Mature stage II

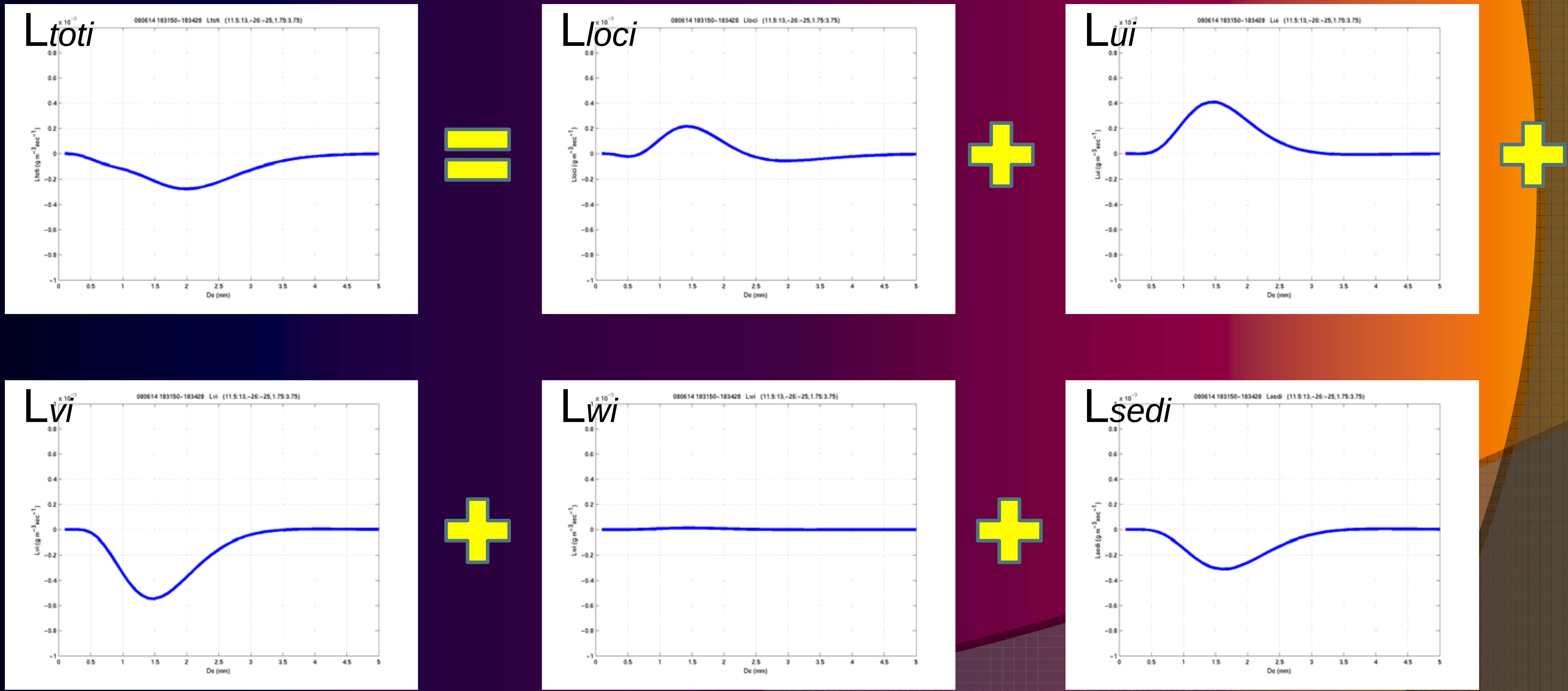




Results

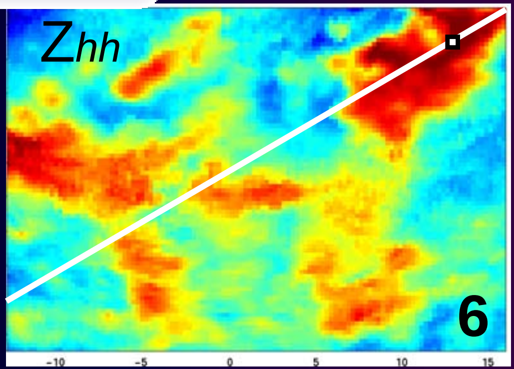
Budget analysis 2703 **3150 3428** 3925 Dissipating stage

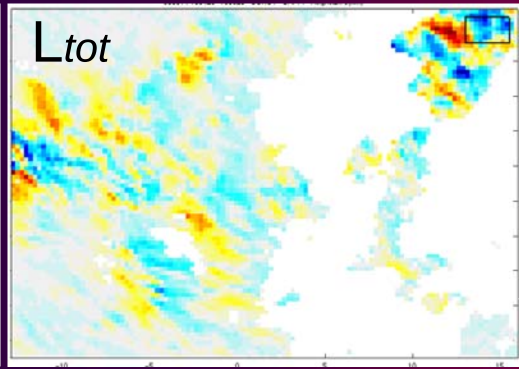


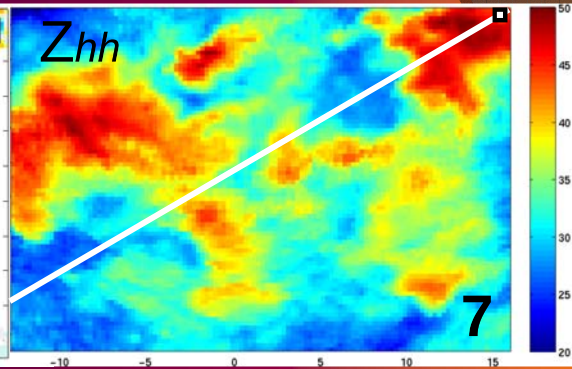


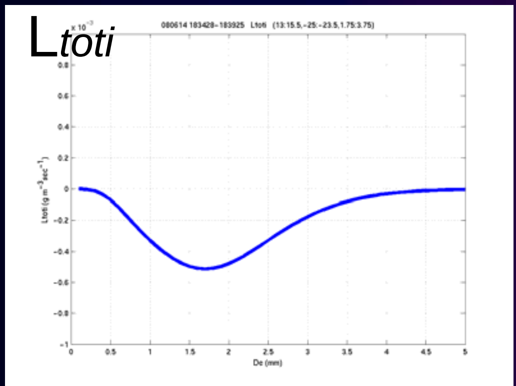
Results

Budget analysis 2703 3150 **3428 3925** Dissipating stage









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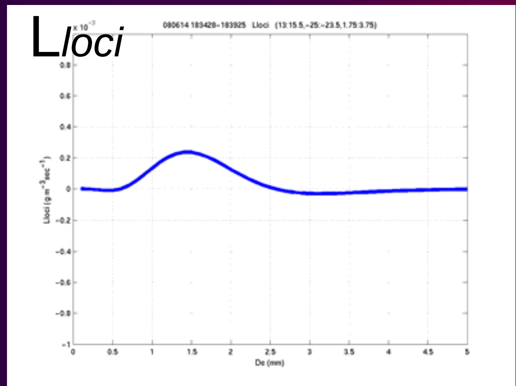

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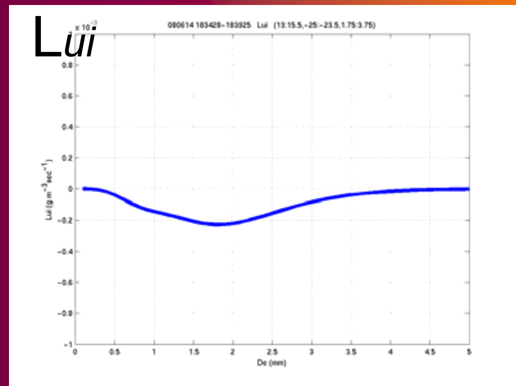

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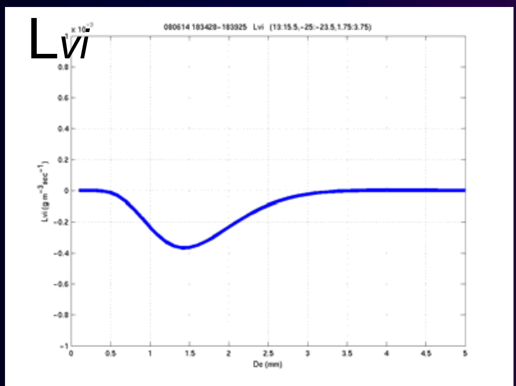

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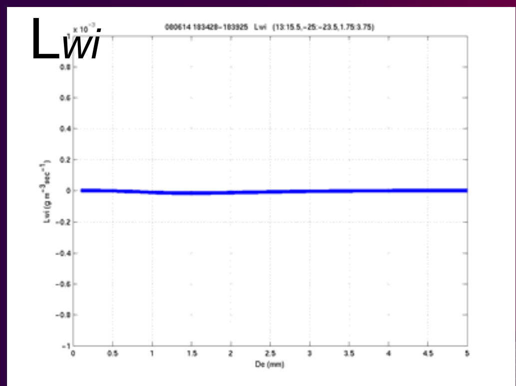

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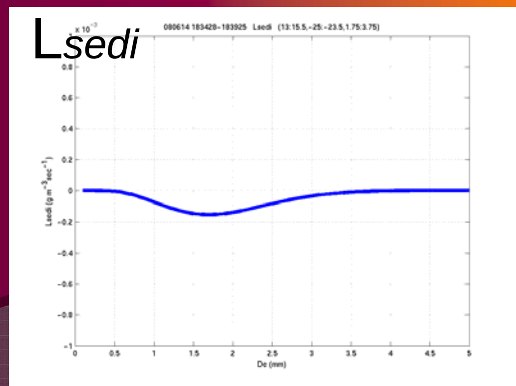


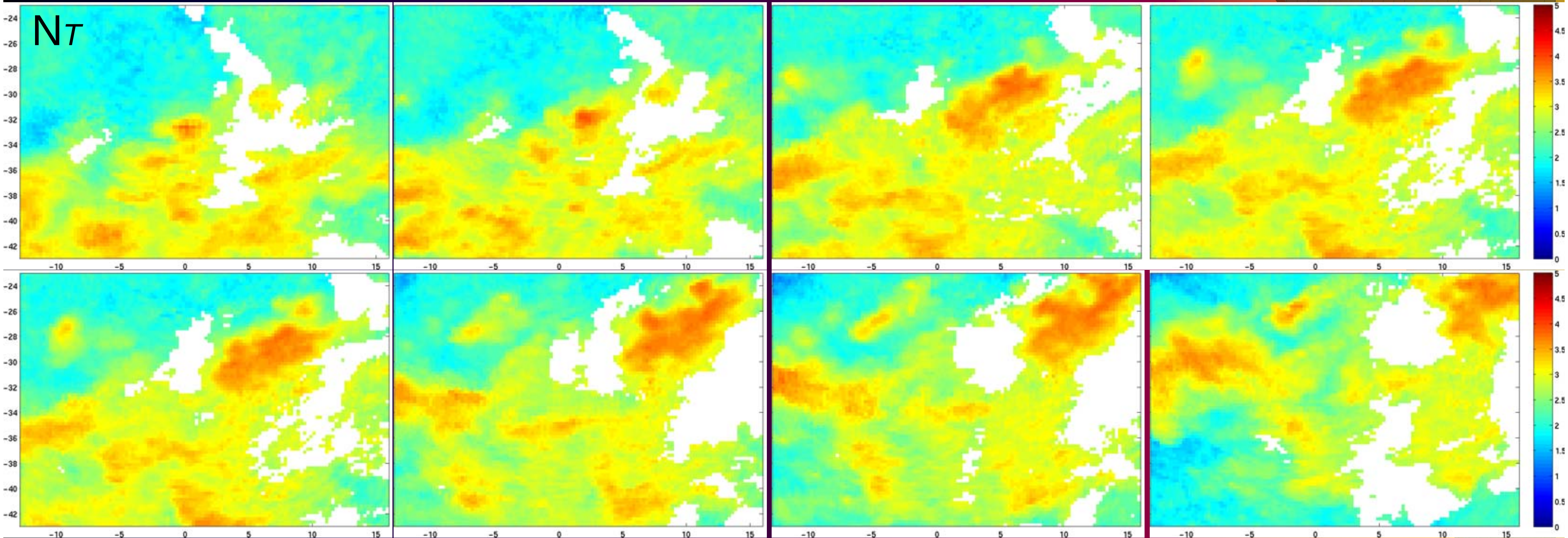
1. A methodology is developed for the budget analysis of drop size distribution using polarimetric radar data.
2. Microphysics inside the warm rain region of the cell:
   (1) Intensifying stage:
   - Drops of all sizes increase.
   Inference: Coalescence and condensation.
   (2) Mature stage I:
   - Drops decrease for $D_e < 0.8$ mm & $D_e > 2.3$ mm.
   - Drops increase for $0.8 < D_e < 2.3$ mm.
   Inference: Coalescence for $D_e < 0.8$ mm & breakup for $D_e > 2.3$ mm.
   (3) Mature stage II:
   - Drops of all sizes almost keep constant.
   Inference: DSD equilibrium.
   (4) Dissipating stage (3150→3428→3925 UTC):
   - Drops of all sizes decrease.
   Inference: Evaporation.
3. The local derivative is positive for all stages, but the total derivative is not! Therefore, we cannot infer what microphysical process occurs without considering advection and sedimentation!

THE END

Thanks for your attention!

1. The data suitable for this research during SoWMEX/TiMREX are insufficient because of the scanning strategies of SPOL and TEAM-R. More intensive sector scans with dual-Doppler wind synthesis are expected in future experiments.

2. To test more cases of different weather phenomena.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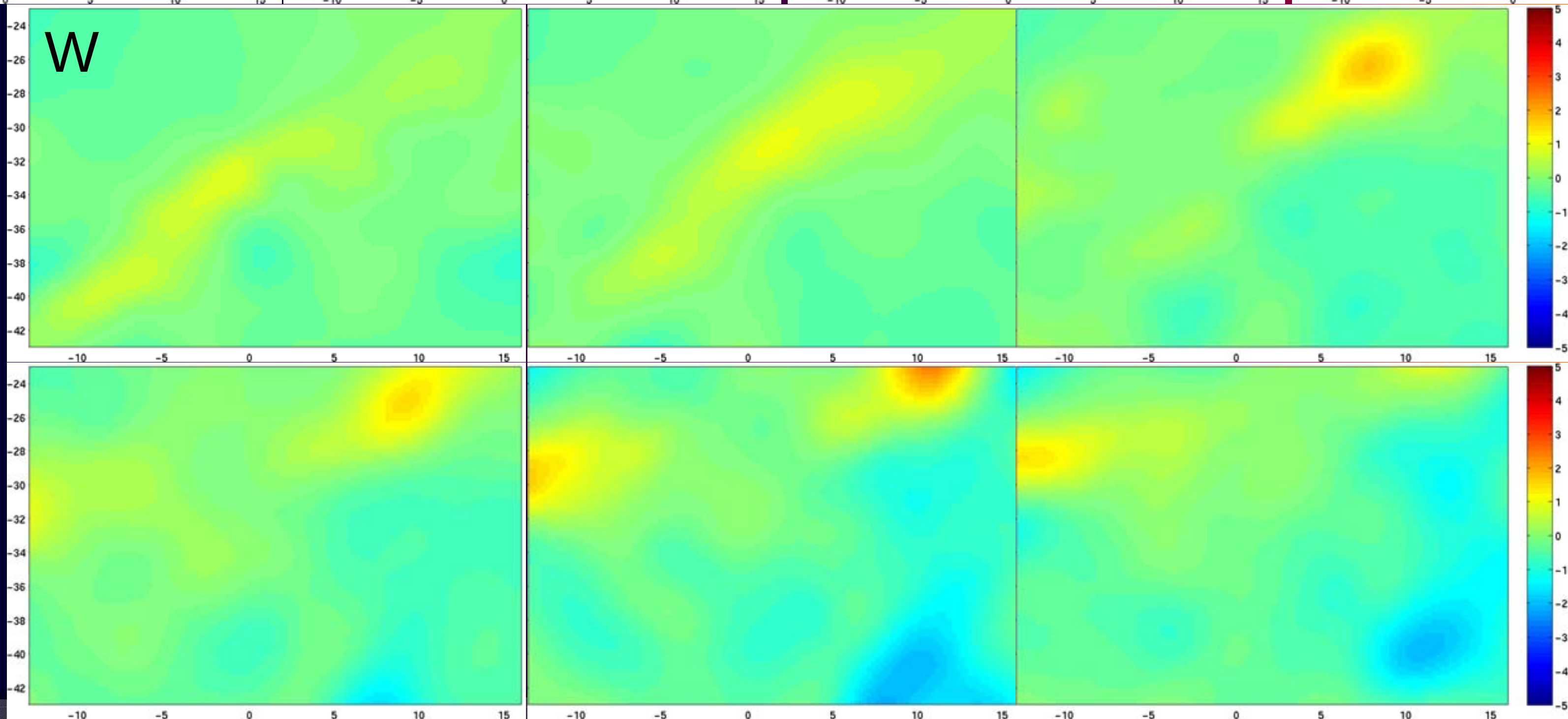
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